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Effect of relevant umklapp process on the two-leg Hubbard ladder with a half-filled band under pressure

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Abstract. By means of perturbative renormalization approach we study the effect of relevant umklapp process on dimensional crossover caused by interladder one particle hopping $t_\perp$ in weakly coupled two-leg Hubbard ladders with a half filled-band. We found that a crossover takes place at a finite value $t_{\perp c}$ which increases as the amplitude of umklapp process increases. For $t_\perp < t_{\perp c}$ the system undergoes a phase transition to the spin density wave phase (SDW) *via* the two particle hopping process, while for $t_\perp > t_{\perp c}$ the system undergoes a crossover to the two dimensional Fermi liquid phase *via* one particle hopping process.

PACS. 64.60.-i General studies of phase transitions – 71.27.+a Strongly correlated electron systems; heavy fermions – 74.25.Dw Superconductivity phase diagrams – 74.72.-h High-T_c compounds

1 Introduction

With the view to understand the behavior of high T_c superconductors, great attention is devoted to low dimensional strongly correlated quantum systems in particular spin ladder compounds especially after the experimental discovery of superconductivity in a doped spin ladder material under pressure [1]. These compounds are considered as an intermediate situation between one and two dimensional systems [2]. Even-leg ladders which are spin liquids exhibit as high-T_c superconductors a spin gap in their excitation spectrum and a transition from insulator to metal upon doping. The study of doping and filling effects in such materials is promising in this regard. The single half-filled Hubbard ladder was already studied [3] and are found to exhibit a Mott insulating phase.

In this paper we will be interested in Hubbard ladders with a half filled band which represents a special case of filling and may generate umklapp scattering within this band. The ladders are under pressure and are weakly coupled *via* one particle hopping process. The effect of umklapp process has been studied in the case of half-filled chain [4]. Using perturbative renormalization group theory (PRG) [5,6], we will discuss the effect of relevant umklapp process on the phase diagram. We will show that the isolated ladder scale to a quasi-one dimensional insulator where a Mott gap opens in the charge excitation spectrum and consequently the spin density wave correlation (SDW) becomes the most dominant. Such behavior is reminiscent of organic conductors with half-filled band [7]. We will study the competition between umklapp scattering and one particle hopping process. Such study has revealed the existence of deconfinement-confinement transition in the case of two coupled half-filled chains [8].

The setup of this paper is as follows: in Section 2 we present the model giving our choice of the coupling constants. The scaling equations for two particle scattering are derived and solved numerically in Section 3. In Section 4 we study the one and two particle hopping processes and depict the phase diagram. Finally, Section 5 is devoted to the conclusion.

2 The model

We start with the noninteracting two chains Hamiltonian

$$H_0 = -t \sum_{i,j=i+1} \sum_{\sigma=\pm 1} \left(c^+_{i,1,\sigma} c_{j,1,\sigma} + c^+_{i,2,\sigma} c_{j,2,\sigma} + H.C. \right) - t' \sum_{i} \sum_{\sigma=\pm 1} \left(c^+_{i,1,\sigma} c_{i,2,\sigma} + H.C. \right), \quad (1)$$

$c^+_{i,1,\sigma} (c^+_{i,2,\sigma})$ creates an electron with spin σ at site i on chain 1(2).

There are two bands labeled A (Antibonding) and B (Bonding) which can be linearized in the form

$$\epsilon^m_k = \pm v^m_{\mathrm{F}} \left(k \mp k^m_{\mathrm{F}} \right), \quad m = \mathrm{A},\ \mathrm{B} \quad (2)$$

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within some cutoff range k_0 [9,10], where $v_F^m = 2t \sin k_F^m$. Consequently ϵ_k^m run over a range characterized by the bandwidth cutoff $E_0 = 2k_0 v_F$ where v_F is the Fermi velocity in absence of t' (see Fig. 1 of Ref. [9] and Fig. 3a of Ref. [6]). We assume that $v_F^m = v_F$ to avoid additional renormalization of the Fermi velocity [11].

The intraladder Hubbard repulsion U generates scattering processes. Let us assume that $4k_F^A$ is equal to a reciprocal lattice vector. Thus umklapp process denoted by $g^{(3)}_{AAAA}$ will contribute to the interacting Hamiltonian. In this case, intraladder processes denoted by $g_0^{(i)}$ by Kishine *et al.* [6] differ on whether the interacting electrons belong to the band A or B. Consequently we obtain nine dimensionless coupling constants denoted by $g^{(3)}_{AAAA}$ and $g^{(i)}_{XYVW}$ where $i = 1, 2$ and X, Y, V and W stand for A and B [11]. $g^{(1)}_{XYVW}$ denote backward scattering whereas $g^{(2)}_{XYVW}$ denote forward scattering. We will consider the case of isotropic spin coupling [9]. For simplicity we denote the $g^{(i)}_{XYVW}$ as follows $g^{(i)}_{AAAA} \equiv g^{(i)}_A$, $g^{(i)}_{BBBB} \equiv g^{(i)}_B$, $g^{(3)}_{AAAA} \equiv g^{(3)}_A$, $g^{(1)}_{ABAB} \equiv g^{(1)}_f$, $g^{(2)}_{ABBA} \equiv g^{(2)}_f$, $g^{(i)}_{AABB} \equiv g^{(i)}_t$.

3 RG equations for two particle scattering

The interladder one particle hopping $t_\perp$ and the coupling constants generate two types of dimensional crossover: one particle crossover and two particle crossover [6]. We use the perturbative renormalization group theory (PRG) to study the competition between the two kinds of dimensional crossover.

To derive the scaling equations for the coupling constants, the multiplicative renormalization group theory (MRG) is used within the g-ology model [10]. In this theory, it is assumed that when decreasing the cutoff E_0 to a value $E_0' = E_0 \mathrm{e}^{-l}$, the vertices and Green's functions get multiplied by factors that depend only on the cutoff scale. We denote by d_m the dimensionless Green's function corresponding to the propagation in the band m. The renormalized Green's function d_m' are given by [11]

$$d_m'\left(\frac{\omega}{E_0'}, g'\right) = z_m\left(\frac{E_0'}{E_0}, g\right) d_m\left(\frac{\omega}{E_0}, g\right) \tag{3}$$

where g' is the renormalized coupling constant.

The dimensionless vertex $\Gamma^{(i)}_{XYVW}$ satisfy the following equation

$$\Gamma'^{(i)}_{XYVW}\left(\frac{\omega}{E_0'}, g'\right) = [z^{(i)}_{XYVW}]^{-1} \Gamma^{(i)}_{XYVW}\left(\frac{\omega}{E_0}, g\right), \quad i = 1, 2; \quad X,\ Y,\ V,\ W = \mathrm{A},\ \mathrm{B} \tag{4}$$

whereas the renormalized coupling g' are given by:

$$g'^{(i)}_{XYVW} = g^{(i)}_{XYVW} z^{(i)}_{XYVW} \left[z_X z_Y z_V z_W\right]^{-\frac{1}{2}} \tag{5}$$

the umklapp vertex $\Gamma^{(3)}_A$ and the coupling constant $g^{(3)}_A$ satisfy similar equations.

We can verify that the following quantity is invariant under scaling transformation [10]

$$g^{(i)}_{XYVW} \Gamma^{(i)}_{XYVW} \left[d_X d_Y d_V d_W\right]^{\frac{1}{2}}. \tag{6}$$

Developing $\Gamma^{(i)}_{XYVW}$ and the Green's functions to the first order in $\mathrm{Log}\frac{\omega}{E_0}$ and using the scaling invariant given in equation (6), we can derive the scaling equations for the coupling constants (given in Appendix A) and Green's functions. These equations are depicted in Figure 1. We have performed numerical integration of the scaling equations. Starting with the following bare coupling

$$g^{(3)}_A = g^{(i)}_\mu = 0.3, \quad \mu = \mathrm{A},\ \mathrm{B},\ f,\ t; \quad i = 1, 2 \tag{7}$$

the scaling equations lead to

$$g^{(1)*}_A = g^{(1)*}_B = 0,\ g^{(2)*}_A = 0.5,\ g^{(3)*}_A = 1, \\ g^{(i)*}_f = g^{(i)*}_t = 0,\ i = 1, 2. \tag{8}$$

In this case the umklapp process becomes relevant and the isolated ladders scale to a phase where the two bands are completely decoupled ($g^{(i)*}_f = g^{(i)*}_t = 0,\ i = 1, 2$). Therefore we could define the stiffness of the charge excitation in the antibonding band in the same manner as for a single chain that is $K_\rho^A = \sqrt{(1+G)/(1-G)}$ where $G = g^{(1)}_A - 2g^{(2)}_A$. The fixed point is then characterized by $K_\rho^A = 0$ which means that a Mott gap opens in the charge excitation spectrum of the A band.

However the bonding band and the spin excitation spectrum of the antibonding band remain gapless. Consequently the SDW channel in the A band becomes the most dominant as will be shown in the next section.

Figure 2 represents the scaling flows for the bare coupling mentioned above.

4 RG equations for one and two particle hopping

In this section we consider that the ladders are weakly coupled by one and two particle hopping processes and we study the effect of such processes on the phase diagram. The one particle hopping (two particle hopping) takes place when a single particle (pair of particle) hops from one ladder to a neighboring one as it is illustrated in Figure 2 of reference [6]. As we have distinguished the intrachain processes in A and B bands we should distinguish between hopping process in the A band, denoted by $t_{\perp A}$, and the one in the B band denoted $t_{\perp B}$.

To derive the scaling equations for the one and the two particle processes we have adopted the renormalization group formulation of Bourbonnais and Caron [5] based on Kadanoff-Wilson (WK) model. This approach was also applied in the case of non half-filled ladder [6]. The multiplicative renormalization group (MRG) method used to derive the renormalization group equations for the coupling constants in Section 3 is enable to generate new relevant couplings such as two particle hopping [5]. However

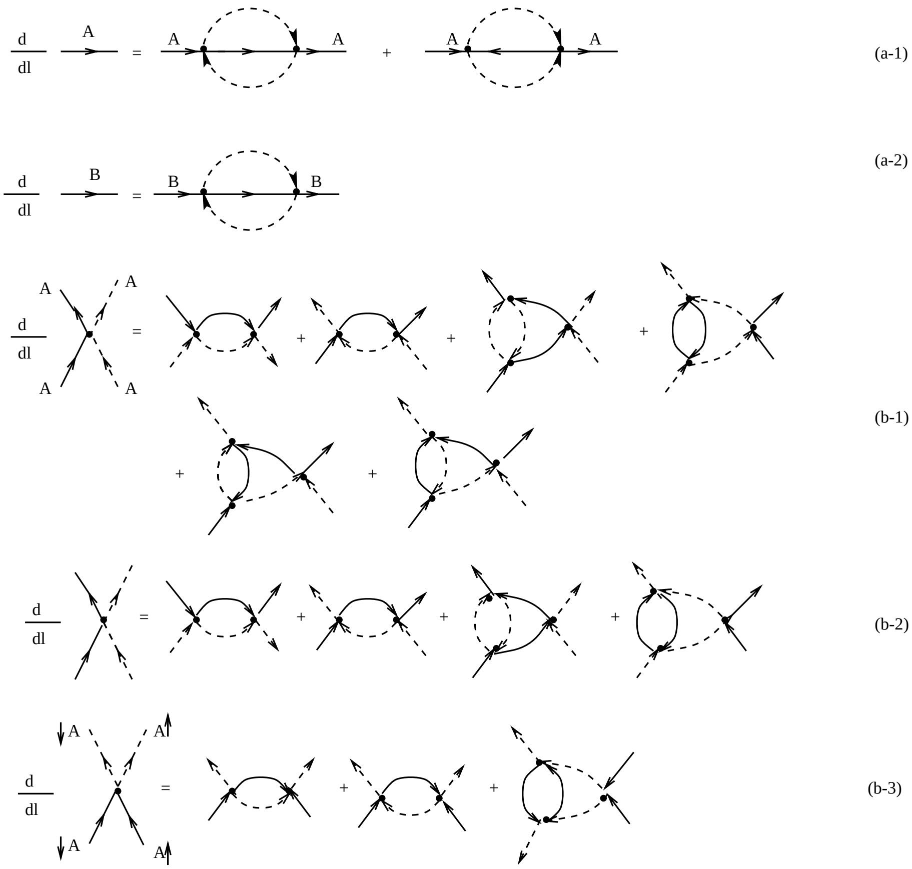

Fig. 1. Diagrammatic representations of the scaling equations for interladder one particle propagator on the band A (a-1), interladder one particle propagator on the band B (a-2), intraladder scattering vertices $\Gamma_{\mathrm{A}}^{(i)}$, $i=1,2$ (b-1), intraladder scattering vertices $\Gamma_{\mathrm{B}}^{(i)}$, $\Gamma_{f}^{(i)}$, $\Gamma_{t}^{(i)}$, $i=1,2$, (b-2), intraladder scattering vertex $\Gamma_{\mathrm{A}}^{(3)}$ (b-3).

the WK renormalization group lead to the same renormalization equations for the $g_{\mu}^{(i)}$ as the MRG. We have used these two methods for diversity. The details of derivation of the one and two particle scaling processes exist already in references [5,6].

The scaling equations for $t_{\perp \mathrm{A}}$ and $t_{\perp \mathrm{B}}$ are then given by (Fig. 3):

$$\frac{\mathrm{d}\ \mathrm{Log}\ \widetilde{t_{\perp \mathrm{A}}}}{\mathrm{d}l} = 1 - \Big(g_{\mathrm{A}}^{(1)2} + g_{\mathrm{A}}^{(2)2} - g_{\mathrm{A}}^{(1)} g_{\mathrm{A}}^{(2)} + g_{f}^{(1)2} + g_{f}^{(2)2} - g_{f}^{(1)} g_{f}^{(2)} + g_{t}^{(1)2} + g_{t}^{(2)2} - g_{t}^{(1)} g_{t}^{(2)} + \frac{g_{\mathrm{A}}^{(3)2}}{2}\Big) \tag{9}$$

$$\frac{\mathrm{d}\ \mathrm{Log}\ \widetilde{t_{\perp \mathrm{B}}}}{\mathrm{d}l} = 1 - \Big(g_{\mathrm{B}}^{(1)2} + g_{\mathrm{B}}^{(2)2} - g_{\mathrm{B}}^{(1)} g_{\mathrm{B}}^{(2)} + g_{f}^{(1)2} + g_{f}^{(2)2} - g_{f}^{(1)} g_{f}^{(2)} + g_{t}^{(1)2} + g_{t}^{(2)2} - g_{t}^{(1)} g_{t}^{(2)}\Big) \tag{10}$$

where $\widetilde{t}_{\perp m}(l) = \frac{t_{\perp m}(l)}{E_0}$.

The bare value $\widetilde{t}_{\perp m}(0) \equiv \widetilde{t}_{\perp 0}$ may be regarded as an applied pressure [6]. The bare coupling and the fixed point given respectively by equations (7, 8). In this case the one particle processes become relevant and the interladder two particle processes are dominated by the intraband spin density wave channel in the band A. In fact in the A band the Mott gap locks the CDW and the superconducting channels whereas the SDW channel, enhanced by the umklapp process, becomes

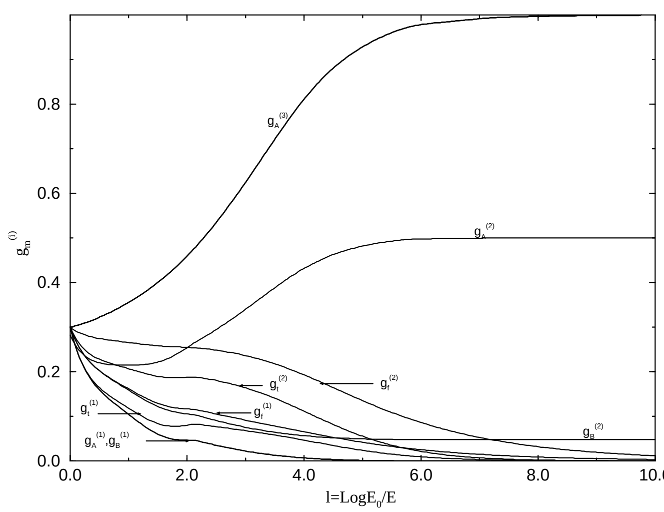

Fig. 2. Scaling flows of the coupling constants for relevant umklapp process.

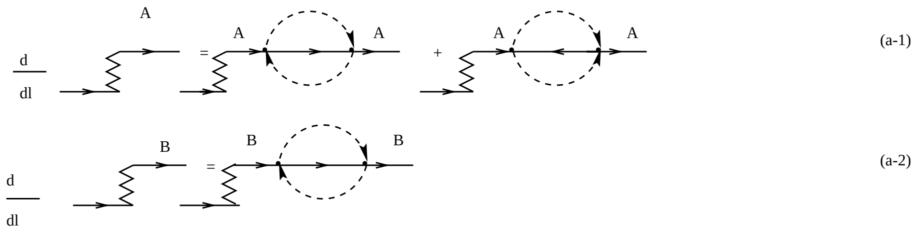

Fig. 3. Diagrammatic representations of the scaling equations for the interladder one particle hopping in the band A, $\widetilde{t}_{\perp\mathrm{A}}$, (a-1) and in the band B, $\widetilde{t}_{\perp\mathrm{B}}$, (a-2).

the most dominant. However in the bonding band, although the spin and charge excitation modes remain gapless, the amplitudes of the different channels are negligible compared to the amplitude of the SDW channel's generator in the antibonding band denoted by $V_{\mathrm{A}}^{\mathrm{SDW}}$. This is due to the absence of umklapp process in the B band and the weakness of the coupling constants to which the ladders scale. The enhancement of the SDW correlation by umklapp process was also found in the quasi-one dimensional organic conductors $(\mathrm{TMTSM})_2\mathrm{X}$, $(\mathrm{X} = \mathrm{ClO}_4, \mathrm{PF}_6)$ which may be regarded as coupled chains due to strong anisotropy [12,13].

The scaling equation of the SDW generator $V_{\mathrm{A}}^{\mathrm{SDW}}$ in the band A depend on the amplitude of the umklapp generator V^{um} [4]. The scaling equations are given by:

$$\frac{\mathrm{d}V_{\mathrm{A}}^{\mathrm{SDW}}(l)}{\mathrm{d}l} = -\frac{1}{4}\,\widetilde{t}_{\perp\mathrm{A}}^{\,2}\,(g_{\mathrm{A}}^{(2)2} + 4g_{\mathrm{A}}^{(3)2}) + g_{\mathrm{A}}^{(2)}V_{\mathrm{A}}^{\mathrm{SDW}} + 4g_{\mathrm{A}}^{(3)}V^{\mathrm{um}} - \frac{1}{2}\left[(V_{\mathrm{A}}^{\mathrm{SDW}})^2 + 4(V^{\mathrm{um}})^2\right] \tag{11}$$

$$\frac{\mathrm{d}V^{\mathrm{um}}(l)}{\mathrm{d}l} = -\,\widetilde{t}_{\perp\mathrm{A}}^{\,2}\,g_{\mathrm{A}}^{(2)}g_{\mathrm{A}}^{(3)} + 4\left(g_{\mathrm{A}}^{(2)}V^{\mathrm{um}} + g_{\mathrm{A}}^{(3)}V_{\mathrm{A}}^{\mathrm{SDW}}\right) - 2V_{\mathrm{A}}^{\mathrm{SDW}}V^{\mathrm{um}}. \tag{12}$$

Henceforth we will denote $V_{\mathrm{A}}^{\mathrm{SDW}}$ by V^{SDW}.

It is worth noting that in the case of irrelevant umklapp process [14], the d-wave superconducting correlation remains the most dominant as in the case of non half-filled ladder [6] and there is no important change in the phase diagram of reference [6].

We have solved numerically the scaling equations (A1–A9, 9–12).

Figure 4 shows the scaling flows of $\widetilde{t}_{\perp m}\,(l)$ and $V^{\mathrm{SDW}}(l)$ for $\widetilde{t}_{\perp 0} = 0.04$ and $\widetilde{t}_{\perp 0} = 0.12$.

As we can see $\widetilde{t}_{\perp\mathrm{A}}(l)$ grows much slowly than $\widetilde{t}_{\perp\mathrm{B}}(l)$ which reminds us the case of half-filled chains [4] where the growth of the interchain umklapp process reduces the growth of $\widetilde{t}_{\perp}$.

For $\widetilde{t}_{\perp 0} = 0.12$, $\widetilde{t}_{\perp\mathrm{B}}(l)$ reaches unity before $V^{\mathrm{SDW}}(l)$ diverges. Thus the one particle crossover dominates the two particle crossover. In this case the system crosses over to a two dimensional phase.

However, for $\widetilde{t}_{\perp 0} = 0.04$, $V^{\mathrm{SDW}}(l)$ diverges before $\widetilde{t}_{\perp\mathrm{B}}\,(l)$ reaches unity, so a two particle crossover takes place.

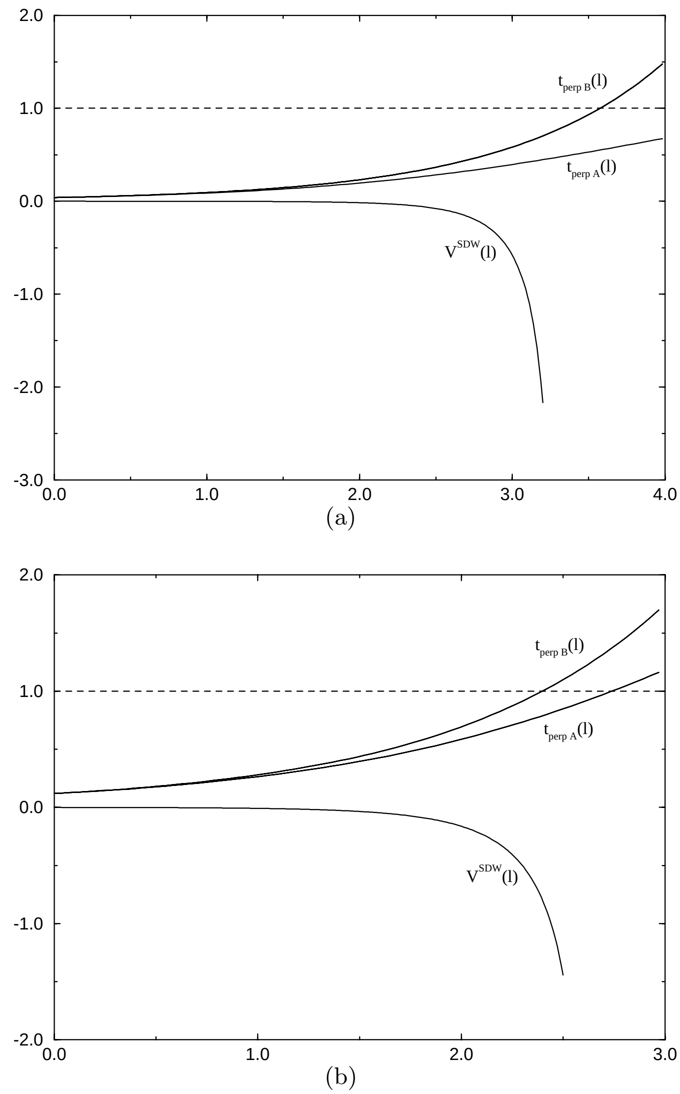

Fig. 4. Scaling flows of $\widetilde{t}_{\perp\mathrm{A}}(l)$, $\widetilde{t}_{\perp\mathrm{B}}(l)$ (denoted by $\widetilde{t}_{\perp\mathrm{A}}$ and $\widetilde{t}_{\perp\mathrm{B}}$ resp.) and $V^{\mathrm{SDW}}(l)$, for $\widetilde{t}_{\perp 0} = 0.04$ (a) and for $\widetilde{t}_{\perp 0} = 0.12$ (b).

In the same manner as in reference [6] we define a crossover temperature $\widetilde{T}_{\mathrm{cross}}$ and the SDW temperature $\widetilde{T}_{\mathrm{SDW}}$ by

$$\widetilde{T}_{\mathrm{cross}} = \frac{T_{\mathrm{cross}}}{E_0} = \mathrm{e}^{-l_{\mathrm{cross}}}, \quad \widetilde{T}_{\mathrm{SDW}} = \frac{T_{\mathrm{SDW}}}{E_0} = \mathrm{e}^{-l_{\mathrm{SDW}}} \tag{13}$$

where $\widetilde{t}_{\perp}(l_{\mathrm{cross}}) = 1$ and $V^{\mathrm{SDW}}(l_{\mathrm{SDW}}) = -\infty$.

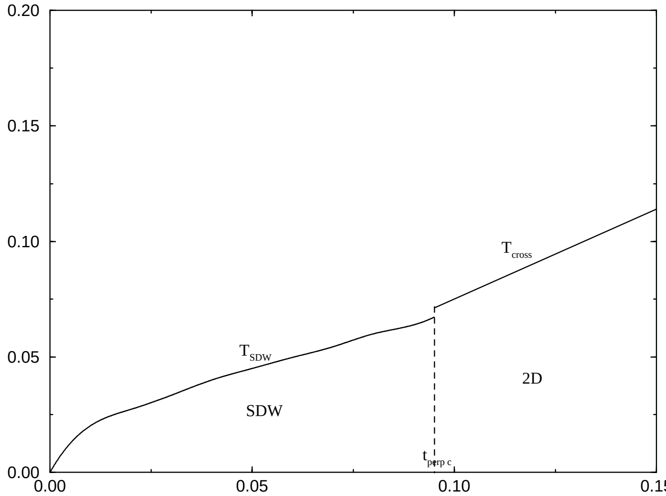

Fig. 5. Phase diagram of weakly coupled Hubbard ladder with a half-filled band in presence of relevant umklapp process.

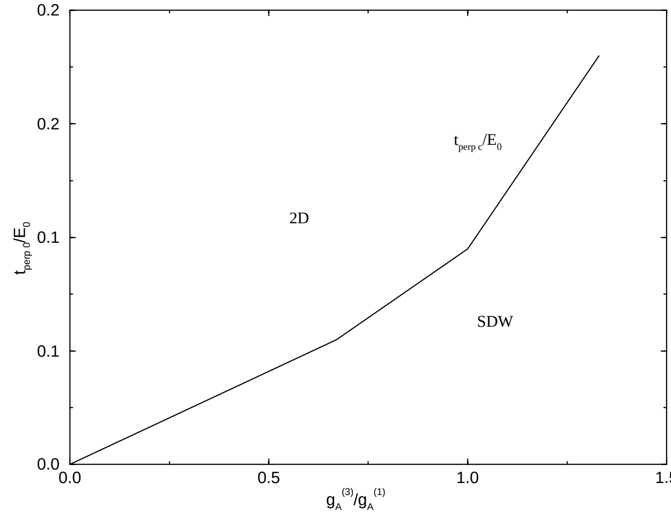

Fig. 6. Effect of relevant umklapp process on the critical value $\widetilde{t}_{\perp c}$ for $g_\mu^{(i)} = 0.3$, $\mu = \mathrm{A}, \mathrm{B}, f, t$; $i = 1, 2$ ($\widetilde{t}_{\perp c}$ and $\widetilde{t}_{\perp 0}$ are denoted by $\widetilde{t}_{\perp c}\ /E_0$ and $\widetilde{t}_{\perp 0}\ /E_0$ resp.).

We should note that the value of $\widetilde{T}_{\mathrm{SDW}}$ and $\widetilde{T}_{\mathrm{cross}}$ are disputable especially when l_{SDW} or l_{cross} are in the region where the coupling constants reach their fixed point value. In such case the system scales to a strong coupling region where the RG treatment is no more reliable because it is based on weak coupling assumption. However the RG method provide a qualitative description of the behavior of the system. A more accurate treatment may be the bosonization method.

To obtain the phase diagram depicted in Figure 5, we have integrated the scaling equations for different value of $\widetilde{t}_{\perp 0}$ with the bare coupling already mentioned. We note that there is a finite value $\widetilde{t}_{\perp c} = 0.095$ for $\widetilde{t}_{\perp}$ where a dimensional crossover takes place.

For $\widetilde{t}_{\perp 0} < \widetilde{t}_{\perp c}$, and as the temperature decreases, the system undergoes a phase transition, *via* two particle hopping, to the SDW phase at $\widetilde{T}_{\mathrm{SDW}}$ that increases with increasing $\widetilde{t}_{\perp 0}$ which may be regarded as an applied pressure [4].

In this range of temperature and pressure the one particle hopping process is confined within the ladder and the Mott localization inhibits the pair hopping process within CDW and superconducting channels. The confinement of the one particle hopping may results of the incoherence of the interladder propagation rather than the irrelevance of hopping process which remains relevant under scaling [15]. We should however note that in the case of two coupled chains with umklapp scattering it is found that the interchain hopping becomes irrelevant which yields to deconfinement-confinement transition [8].

For $\widetilde{t}_{\perp 0} > \widetilde{t}_{\perp c}$ the one particle hopping process bypasses the SDW correlation and the system crosses to the two dimensional phase at $\widetilde{T}_{\mathrm{cross}}$.

The dependence of $\widetilde{t}_{\perp c}$ on the amplitude of the umklapp process is depicted in Figure 6. We see that as the umklapp scattering becomes important the SDW phase gets wider and the one particle process is strongly suppressed. This behavior is also found in the case of a system of coupled half-filled chains [4].

The phase diagram shown in Figure 5 is reminiscent of the one found in the case of quasi-one dimensional organic conductors $(\mathrm{TMTTF})_2\mathrm{X}$ and $(\mathrm{TMTSF})_2\mathrm{X}$ (Fig. 1 of Ref. [12]) if we disregard the spin Peierls phase which is related to the lattice distortion. Therefore, a system of two-leg Hubbard ladders with a half-filled band may behave as organic conductors. This may be confirmed by the study of the two dimensional Fermi liquid phase (Fig. 5) using the renormalization group approach for a two dimensional system as in reference [12] in order to seek the dominant phase which may depend on the structure of the Fermi surface and nesting conditions. Such study gets over the scope of this paper.

This remark reflects the fact that ladder materials represent an intermediate situation between one and two dimensional systems and acting on the filling, doping etc., will tip up the ladder to the one or two dimensional behavior.

It is worth to note that isolated non half-filled chains scale to a weak coupling Tomonaga-Luttinger gapless phase and the interchain one particle process becomes relevant. The coupling chains always undergo a crossover to a two dimensional phase. However, half-filled isolated chains scale to a phase where a Mott gap opens and in this case, such as the case of the one half-filled band ladder discussed above, the crossover takes place at a finite value of $t_\perp$ below which the two particle process dominates the one particle process although $t_\perp$ is relevant under scaling.

5 Conclusion

In this paper we have studied weakly coupled two-leg Hubbard ladders with a half-filled band. The ladders are weakly coupled *via* one particle hopping process $\tilde{t}_{\perp 0}$. We have considered a model with a linear dispersion relation near Fermi points. By means of the renormalization group approach we have studied the effect of relevant umklapp process on the dimensional crossover caused by one particle process. We have found that the isolated ladder scale to a quasi-one dimensional Mott insulator where the SDW correlation is the most dominant.

We have shown that a dimensional crossover between spin density wave phase (SDW) and a two dimensional Fermi liquid phase (2D) takes place at a finite value $\widetilde{t}_{\perp c}$ that increases with increasing umklapp process which makes the SDW phase wider.

For $\widetilde{t}_{\perp 0} < \widetilde{t}_{\perp c}$, a transition to the SDW phase takes place whereas the one particle hopping process is found to be confined within the ladder.

However for $\widetilde{t}_{\perp 0} > \widetilde{t}_{\perp c}$, the one particle process becomes the most relevant and a crossover to 2D phase occurs.

It should be noted that this behavior is reminiscent of the one found in the case of quasi-one dimensional organic conductors. This may be confirmed by the properties of the 2D phase which may be obtained within the renormalization group theory applied to a two dimensional system. This shows the relevance of ladder materials which will permit to make the interpolation between one and two dimensional systems and may be of great importance to understand the behavior of high-T_c superconductors.

Compared to the case of half-filled chains, ladders with a half-filled band and relevant umklapp process have the same behavior. In both cases there is a Mott gap in the charge excitation spectrum and the SDW correlation becomes the most dominant. The crossover form SDW phase to the two dimensional phase takes place at finite value of $t_\perp$ although $t_\perp$ remains relevant under scaling. However in non half-filled chains, the system always undergoes a phase transition to a two dimensional phase for any value of $t_\perp$ and under scaling $t_\perp$ grows more rapidly than in the case of half-filled chain.

Our study predicts the behavior of two-leg Hubbard ladder system with a half-filled band under pressure. To our knowledge an experimental realization of such material does not exist yet. It is worth noting that we have not treated the case of two-leg Hubbard ladder at half-filling (where the two bands are half-filled) because such material is an insulator [16], while in this paper we are interested in doped ladder in order to understand their experimental behavior [1].

We are grateful to J. Kishine, M. Héritier, C. Bourbonnais and S. Kaddour for stimulating discussions. We acknowledge the referee for interesting remarks. S. Haddad would like to thank le Centre de Recherche en Physique du Solide à l'Université de Sherbrooke for hospitality.

Appendix A: RG equations of the coupling constants

Using the invariant given in equation (6), we have derived the RG equations of the coupling constants. These equations are the following

$$\frac{\mathrm{d}g_{\mathrm{A}}^{(1)}}{\mathrm{d}\,\mathrm{Log}\,x} = 2g_{\mathrm{A}}^{(1)2} + 2g_t^{(1)}g_t^{(2)} + 2g_{\mathrm{A}}^{(1)}\left(g_{\mathrm{A}}^{(1)2} + g_f^{(1)2} + g_t^{(1)2}\right) - 2\left(g_{\mathrm{A}}^{(1)} - g_f^{(1)}\right)g_t^{(2)}\left(g_t^{(1)} - g_t^{(2)}\right) \tag{A.1}$$

$$\frac{\mathrm{d}g_{\mathrm{A}}^{(2)}}{\mathrm{d}\,\mathrm{Log}\,x} = g_{\mathrm{A}}^{(1)2} + g_t^{(1)2} + g_t^{(2)2} - g_{\mathrm{A}}^{(3)2} + g_{\mathrm{A}}^{(1)3} + g_{\mathrm{A}}^{(1)}g_f^{(1)2} + g_f^{(1)}g_t^{(1)2} - g_{\mathrm{A}}^{(3)2}\left(g_{\mathrm{A}}^{(1)} - 2g_{\mathrm{A}}^{(2)}\right) + 2\left(g_{\mathrm{A}}^{(2)} - g_f^{(2)}\right)\left(g_t^{(1)2} - g_t^{(1)}g_t^{(2)} + g_t^{(2)2}\right) \tag{A.2}$$

$$\frac{\mathrm{d}g_{\mathrm{B}}^{(1)}}{\mathrm{d}\,\mathrm{Log}\,x} = 2g_{\mathrm{B}}^{(1)2} + 2g_t^{(1)}g_t^{(2)} + 2g_{\mathrm{B}}^{(1)}\left(g_{\mathrm{B}}^{(1)2} + g_f^{(1)2} + g_t^{(1)2}\right) - 2(g_{\mathrm{B}}^{(1)} - g_f^{(1)})g_t^{(2)}(g_t^{(1)} - g_t^{(2)}) \tag{A.3}$$

$$\frac{\mathrm{d}g_{\mathrm{B}}^{(2)}}{\mathrm{d}\,\mathrm{Log}\,x} = g_{\mathrm{B}}^{(1)2} + g_t^{(1)2} + g_t^{(2)2} + g_{\mathrm{B}}^{(1)3} + g_{\mathrm{B}}^{(1)}g_f^{(1)2} + g_f^{(1)}g_t^{(1)2} + 2\left(g_{\mathrm{B}}^{(2)} - g_f^{(2)}\right)\left(g_t^{(1)2} - g_t^{(1)}g_t^{(2)} + g_t^{(2)2}\right) \tag{A.4}$$

$$\frac{\mathrm{d}g_f^{(1)}}{\mathrm{d}\,\mathrm{Log}\,x} = 2g_f^{(1)2} + 2g_t^{(1)}\left(g_t^{(1)} - g_t^{(2)}\right) + g_f^{(1)} \times\left(2g_f^{(1)2} + g_{\mathrm{A}}^{(1)2} + g_{\mathrm{B}}^{(1)2} + 2g_t^{(1)2} + \frac{1}{2}g_{\mathrm{A}}^{(3)2}\right) - 2g_f^{(1)}g_t^{(2)}\left(g_t^{(1)} - g_t^{(2)}\right) + \left(g_{\mathrm{A}}^{(1)} + g_{\mathrm{B}}^{(1)}\right)g_t^{(2)}\left(g_t^{(1)} - g_t^{(2)}\right) \tag{A.5}$$

$$\frac{\mathrm{d}g_f^{(2)}}{\mathrm{d}\,\mathrm{Log}\,x} = g_f^{(1)2} - g_t^{(2)2} + g_f^{(1)3} + \frac{1}{2}g_t^{(1)2}\left(g_{\mathrm{A}}^{(1)} + g_{\mathrm{B}}^{(1)}\right) + \frac{1}{2}g_f^{(2)}g_{\mathrm{A}}^{(3)2} + 2g_f^{(2)}\left[g_t^{(1)2} - g_t^{(1)}g_t^{(2)} + g_t^{(2)2}\right] - \left(g_{\mathrm{A}}^{(2)} + g_{\mathrm{B}}^{(2)}\right)\left[g_t^{(1)2} - g_t^{(1)}g_t^{(2)} + g_t^{(2)2}\right] + \frac{1}{2}g_f^{(1)}\left(g_{\mathrm{A}}^{(1)2} + g_{\mathrm{B}}^{(1)2}\right) \tag{A.6}$$

$$\frac{\mathrm{d}g_t^{(1)}}{\mathrm{d}\ \mathrm{Log}\, x} = g_t^{(1)}\left(g_{\mathrm{A}}^{(2)} + g_{\mathrm{B}}^{(2)}\right) + g_t^{(2)}\left(g_{\mathrm{A}}^{(1)} + g_{\mathrm{B}}^{(1)}\right)$$
$$+ 2g_t^{(1)}\left(g_f^{(1)} - g_f^{(2)}\right) + 2g_f^{(1)}\left(g_t^{(1)} - g_t^{(2)}\right)$$
$$+ g_t^{(1)}\left(g_{\mathrm{A}}^{(2)} - g_f^{(2)}\right)^2 + g_t^{(1)}\left(g_{\mathrm{B}}^{(2)} - g_f^{(2)}\right)^2$$
$$- g_t^{(1)}\left[\left(g_{\mathrm{A}}^{(1)} - g_f^{(1)}\right)\left(g_{\mathrm{A}}^{(2)} - g_f^{(2)}\right)\right.$$
$$\left.+ \left(g_{\mathrm{B}}^{(1)} - g_f^{(1)}\right)\left(g_{\mathrm{B}}^{(2)} - g_f^{(2)}\right)\right]$$
$$+ g_t^{(1)}\left[g_{\mathrm{A}}^{(1)2} + g_{\mathrm{B}}^{(1)2} + 2g_f^{(1)2} + \frac{1}{2}g_{\mathrm{A}}^{(3)2}\right]$$
$$+ 2g_t^{(1)}\left[g_t^{(1)2} + g_t^{(2)2} - g_t^{(1)}g_t^{(2)}\right] \quad \text{(A.7)}$$

$$\frac{\mathrm{d}g_t^{(2)}}{\mathrm{d}\ \mathrm{Log}\, x} = g_t^{(2)}(g_{\mathrm{A}}^{(2)} + g_{\mathrm{B}}^{(2)} - 2g_f^{(2)}) + g_t^{(1)}\left(g_{\mathrm{A}}^{(1)} + g_{\mathrm{B}}^{(1)}\right)$$
$$+ g_f^{(1)}g_t^{(1)}\left(g_{\mathrm{A}}^{(1)} + g_{\mathrm{B}}^{(1)}\right) + \frac{1}{2}g_t^{(2)}g_{\mathrm{A}}^{(3)2}$$
$$+ g_t^{(2)}\left[\left(g_{\mathrm{A}}^{(1)} - g_f^{(1)}\right)^2\right] + g_t^{(2)}$$
$$\times\left[\left(g_{\mathrm{B}}^{(1)} - g_f^{(1)}\right)^2 - \left(g_{\mathrm{A}}^{(1)} - g_f^{(1)}\right)\left(g_{\mathrm{A}}^{(2)} - g_f^{(2)}\right)\right.$$
$$\left.- \left(g_{\mathrm{B}}^{(1)} - g_f^{(1)}\right)\left(g_{\mathrm{B}}^{(2)} - g_f^{(2)}\right) + \left(g_{\mathrm{A}}^{(2)} - g_f^{(2)}\right)^2\right]$$
$$+ g_t^{(2)}\left[\left(g_{\mathrm{B}}^{(2)} - g_f^{(2)}\right)^2\right]$$
$$+ 2g_t^{(2)}\left[g_t^{(1)2} + g_t^{(2)2} - g_t^{(1)}g_t^{(2)}\right] \quad \text{(A.8)}$$

$$\frac{\mathrm{d}g_{\mathrm{A}}^{(3)}}{\mathrm{d}\ \mathrm{Log}\, x} = 2g_{\mathrm{A}}^{(3)}\left(g_{\mathrm{A}}^{(1)} - 2g_{\mathrm{A}}^{(2)}\right) + 2g_{\mathrm{A}}^{(3)}$$
$$\times\left[g_t^{(1)2} + g_t^{(2)2} - g_t^{(1)}g_t^{(2)} + \frac{1}{2}g_{\mathrm{A}}^{(3)2}\right]$$
$$+ g_{\mathrm{A}}^{(3)}\left[\left(g_{\mathrm{A}}^{(1)} - 2g_{\mathrm{A}}^{(2)}\right)^2 + \left(g_f^{(1)} - 2g_f^{(2)}\right)^2\right] \quad \text{(A.9)}$$

with $x = E_0'/E_0 = \mathrm{e}^{-l}$.

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